Quantum Approximate Optimization of the Long-Range Ising Model with a Trapped-Ion Quantum Simulator

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Abstract

Quantum state preparation on near-term quantum hardware can benefit from a hybrid approach, where a classical computer optimizes a parameterized quantum circuit using energy samples from a quantum device that implements the circuit. The Quantum Approximate Optimization Algorithm (QAOA) framework may be well-suited to testing such a quantum-classical interplay, and has been successful in producing fast, approximate optimization protocols. Here, we apply QAOA to the state preparation problem for a transverse field Ising model (TFIM) with long-ranged, power law interactions. We show that a variationally optimized, low-depth QAOA circuit can scalably approximate the ground state at criticality, and present a novel approach to discovering optimal QAOA paths based on numerical evidence for an asymptotically optimal path. We demonstrate pur approximate state preparation protocol on a trapped ion hardware of up to 40 spins. (arXiv: 1906.02700)

TFIM with power-law coupling

The following model can be simulated on trapped-ion quantum hardware, for up to 40 ions:

$$H(J,B,\alpha) = J\underbrace{\sum_{i<j}\frac{1}{|j-i|^{\alpha}}Z_iZ_j}_{C} + B\underbrace{\sum_i X_i}_{B}$$

Goal: Prepare the ground state of H.

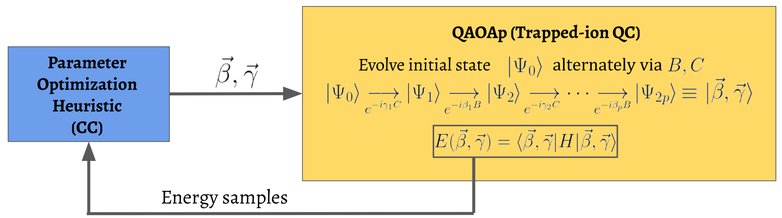
State preparation protocols may use the above Hamiltonians or either components B,C, with evolution times limited by the experiment.

Options:

1. Adiabatic Evolution: Expected to scale poorly at model criticality, due to closing gap.
2. Classical Simulation methods: No guarantees on easy/hardness, but likely hard in general. DMRG works only for a 1D system.
3. (This work) Variational techniques: parameterized circuit guided by a classical search - here, we try QAOA. *A priori, we do not expect this approach to fail in higher dimensional models at criticality.*

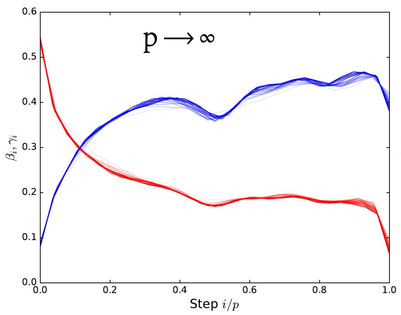
Quantum Approximate Optimization Algorithm

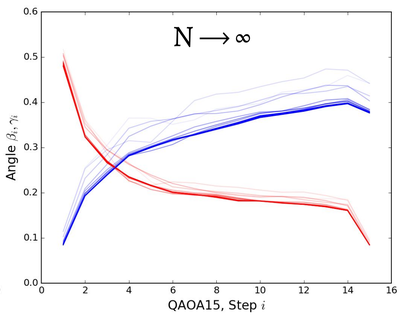
While originally an algorithm for finding approximate solutions to hard combinatorial optimization problems, the QAOA framework is applicable to the problem of state preparation as well.

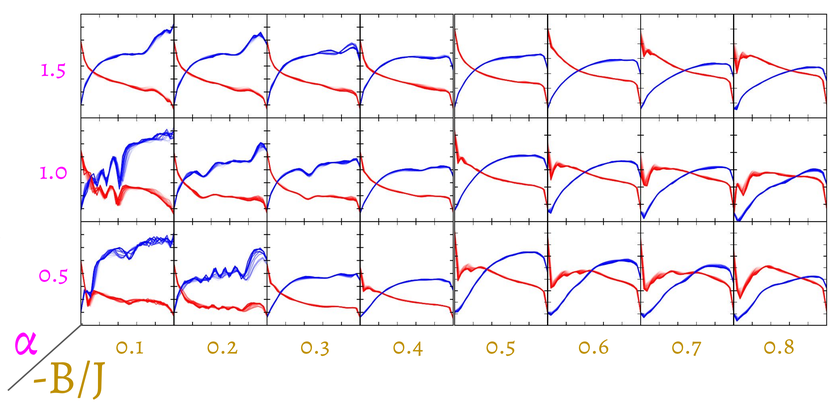


Optimal angle parameters

Empirically, the optimal QAOA angles lie along a smooth curve parameterized by the (normalized) step number, s := (i-1)/(p-1). The angle sets (β_1, β_2, ..., β_p) and (γ_1, γ_2, ..., γ_p) each approximate a curve that converges in the infinite p limit. We see a similar convergence in number of spins N.







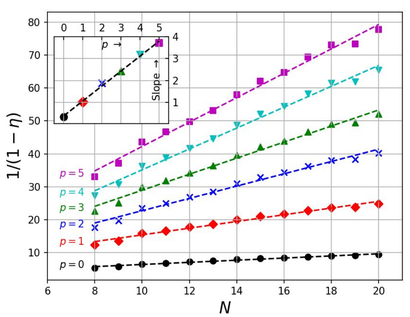
Variational Optimization Heuristic

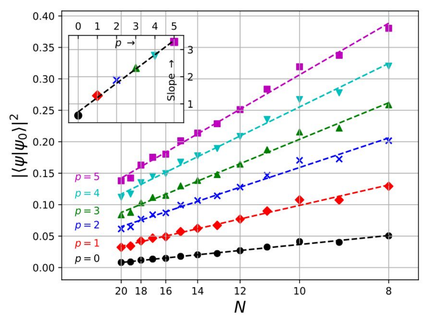
The structure in the QAOA optimal angle sequences can be used to build a scalable heuristic for state preparation:

1. Find angle sequences for low p, N classically.
2. Use dynamic programming in p,N as follows:
   a. N → N+1: For each i=1,...,p, extrapolate the sequence $\{\beta_i^{(m)}: m=1,\ldots,N\}$ (resp. $\{\gamma_i^{(m)}\}$) to generate an initial guess for $\beta_i^{(N+1)}$ (resp. $\gamma_i^{(N+1)}$). Perform local optimization.
   b. p → p+1: Interpolate the optimized angle curve for p, then resample using spacing 1/p to generate p+1 points as an initial guess for local optimization.
3. Terminate at desired p,N.

Our algorithm is expected to scale as a small polynomial in p,N when used with the quantum simulator in a hybrid manner.

Performance Scaling at criticality





Left: η is energy normalized by spectral bandwidth, so that η=0 is the highest energy state while η=1 is the ground state. Numerics suggest that $\eta \sim 1-1/(pN)$ for large p,N.

Right: Squared overlap with the ground state plotted by p,N. The plot suggests that square overlap ~ p/N.

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